Analysis of Layered Social Networks

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Editorial Abstract: The authors present an analytical construct for understanding complex social interactions and relationships. Advanced models may offer decision makers better courses of action when targeting influence, perception, or outcome of actors within a network, through either direct or indirect means.

"To know them means to eliminate them."
From the film Battle of Algiers *[Pontecorvo, 1967].*

Overview

The character suggesting this strategy, Colonel Mathieu, was tasked with quelling the violent insurrection lead by the National Liberation Army, which sought Algerian independence from French rule. The need to understand the adversary's objectives, their underlying social structure, the ebb and flow of social- or other forms of power, consequently placed a greater reliance upon intelligence and analysis, rather than mere application of military force. It could be argued that defeating the elusive terrorist organizations that impose chaos and suffering throughout the world today may require a similar strategy.

Truly knowing these clandestine organizations means understanding how they arrange and build their structures through recruitment, their underlying motivations for violent and seemingly irrational behavior, and their methods of operational control and execution of terrorist activities. Once gained, this knowledge can then be used to identify key individuals, relationships, and organizational practices. Subsequently, such analysis may lead to the identification of weaknesses that can be exploited in an endeavor to either eliminate the network as a whole, cause it to become operationally ineffective, or influence it to directly or indirectly support our own objectives. In today's interconnected world, proficiency in this type of warfare is a necessary condition to ensure US national security, as well as to promote and maintain global stability.

One research area that provides some opportunities to accomplish these goals is social network analysis (SNA). Although the techniques themselves are certainly not new, the notion of their incorporation into the analysis and investigation of non-cooperative organizations (e.g., clandestine, criminal, or terrorist) is of recent interest. In light of the ultimate goal of negating the threat of terrorist networks via the application of influence–ranging from psychological operations to lethal force–SNA provides a means to quantify the roles and promulgation of interpersonal influence. Opportunities to merge the sociological SNA approaches to analysis with those of operations research (OR) and other fields of study have been and should continue to be explored in the context of understanding and taking advantage of organizational phenomena [Hamill, 2006, Clark, 2005, Renfro, 2001].

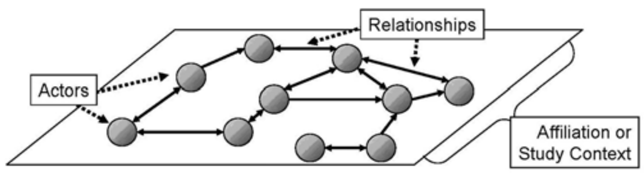

Figure 1: Sociogram of a Notional Social Network

Social Network Analysis

The field of social network analysis is often traced back to the work of Moreno [1953], who developed the sociogram, a pictorial representation of a social group via a graph as shown in Figure 1. Individuals are represented as nodes. Known relationships and their direction, if appropriate, are indicated by arcs. Undirected, or symmetric, relationships imply that the relationship or bond runs equivalently in either direction. If the context of the sociometric data (e.g., accounting for supervisory roles) or if responses within a sociological survey are not equitable between two given actors (e.g., the forgetting of friends), a directed, or asymmetric, arc is more appropriate [cf. Brewer and Webster, 1999].

The majority of SNA measures perform calculations upon the mathematical representation of the sociogram, the sociomatrix (X). The sociomatrix is a two-way, numerical matrix, "indexed by the sending actors (the rows) and the receiving actors (the columns) . . . ," which is equivalent to the adjacency matrix of a graph [Wasserman and Faust, 1994, pg. 77].

Leveraging the natural connection to the mathematically powerful genre of graph theory and its ability to quantitatively study networks, Moreno had devised a means to examine the qualitative nature of relationships among individuals within a social grouping [Moreno, 1953]. Since then, a variety of tools and techniques have been developed to study the structural nature of social networks and the implications of topology (i.e. network structure) and personal characteristics upon overall network behavior. The sociomatrix and the sociological study of networks generally focus upon the two fundamental items of interest within SNA: individual-specific characteristics (such as age, experience, and so forth) that facilitate the attainment of various network positions and the social consequences based upon the actor's position within the social network.

Measures of network position attempt to equate an actor's relative location to the roles they may serve in promulgating, or inhibiting, the flow of influence and information. The most common class of such measures is network centrality.

Variants of centrality such as degree, closeness, and betweenness, measure the connectivity, proximity, or mediating effect of a given individual, respectively [Brass, 1995, pg. 46].

Unfortunately, the majority of existing SNA measures attempting to describe how information, influence, rumors, adoption, and other influences may flow through the network rely only upon a single, presumably dominant, dimension within which the interpersonal relationships have formed. This is primarily due to the belief that the existence of one network (e.g. interpersonal communication) is highly predictive of another (e.g. friendship) [Monge and Contractor, 2003, pg. 296]. This contrasts with the fact that commonly cited characteristics of relationships include frequency of contact, stability of the relationship over time, strength, and multiplexity [Brass, 1995, pg. 45]. These widely accepted concepts or features underlying personal relationships imply that interpersonal relationships may hold more information than a the traditional 'yes or no,' 'is or is not observed,' or 'all or nothing' type of response, and corresponding data, that serves as input to the mathematical models. The question that remains is "Could we be missing critical insights into the behavior, strengths, and weaknesses of social networks by ignoring the multi-dimensionality of human relationships?"

Multiplexity & Layered Networks

Suppose there exist multiple relations pertinent to the evolution of an adversarial network, each measured on the same set of actors. These values ultimately form a 'super-sociomatrix,' offering a means to capture the layers of relations as depicted in Figure 2. Each layer represents a context within which the actors may or may not be affiliated. Contextual examples could include familial relationships, training camps attended together, known friendships, business interactions, known animosities, resources shared, specialized skills or training, and so forth. For each layer or context, if an actor is connected to any other actor in the same context, those actors and that link are recorded. Given that the network of interest is comprised of N individuals, each layer can include no more than the same set of N individuals. Examining the diagram from directly overhead, an actor appearing in more than one context would be aligned vertically, as noted by the 'unique actor' label.

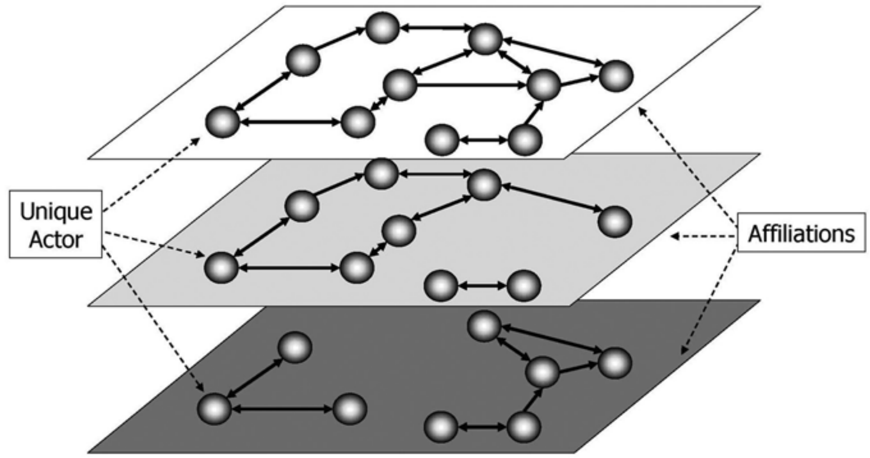

Figure 2: Layered Social Network

Given a set of actors, when more than one relationship or context of interaction is studied the analysis is considered multiplex [Monge and Contractor, 2003, pg. 35]. This term, like many other concepts in SNA, appears to be borrowed from communications theory, which defines multiplex as combining multiple signals into one to facilitate transmission, in such a way that they can later be separated as required [DOD, 2005, pg. 354]. Consequently, communication and interaction between two individuals will generally transmit through several different contexts simultaneously. As Haythornthwaite noted, "we operate in a multiplex world, maintaining multiple roles and relations with others, sustained through a variety of media" [Haythornthwaite, 1999].

Probably the earliest, formal recognition of multi-dimensionality among relationship was described by Granovetter, who suggested that "the degree of overlap of two individuals' friendship networks varies directly with the strength of their tie to one another" [Granovetter, 1973, pg. 1360]. He further defines the strength of a tie as "a (probably linear) combination of the amount of time, the emotional intensity, the intimacy (mutual confiding), and the reciprocal services which characterize the tie" [Granovetter, 1973, pg. 1361]. Ultimately, tie strength assumes that human interaction simultaneously accounts for multiple, underlying relationships or contexts within which those relationships were developed.

For example, two complete strangers may be treated differently based upon the known contexts that comprise a newly formed relationship. Such a case could involve the difference between a random individual standing beside you on a sidewalk and the lady just introduced to you as the new fiancée of your brother. Both are strangers, yet an inevitable difference between the strengths of the two relationships occurs due to the implied trust gained from a familial context. Consequently, it is suggested that by increasing or at least acknowledging the dimensionality of information gathered on individuals of interest, a better understanding of the overall network behavior can be achieved. Determining the contexts or layers of interest is potentially one of the more difficult areas of this problem area, as the types of ties that result in the strong, trusting relationships are likely dependent upon the origins of the organization and the scenario under analysis, or simply predicated upon the available intelligence information.

Relationships that lend themselves to the dimensions of the 'linear combination' serve as strong ties according to Granovetter [1973]. Alternatively, weak ties are composed of casual or intermittent relationships; however, weak ties are potentially strong themselves. The strength of a weak tie lies in its ability to bridge communication or influence between two or more distinct groups, or promote diffusion of influence and ideas between them [Granovetter, 1973, pg. 1363-7]. Clearly, both forms of relationships may prove valuable within a military's investigation of adversary networks.

Of course, how the models (and calculations involved) of such linear combinations should be developed, and even which contexts should be included, remains of theoretical interest [cf. Granovetter, 1973, Marsden and Campbell, 1984, Hite, 2003]. There are a handful of other works that acknowledge the dimensionality of interpersonal relationships and its importance, albeit from a conceptual perspective rather than a mathematical one. For example, Haythornthwaite hypothesized that, in general, the more individuals have in common, the stronger the relationship between them. An article by Friedkin discusses the construction of a Guttman scale—where different stages or assessments can imply others—that incorporates (implicitly equally weighted) dimensions such as 'claims of frequent discussion,' of 'seeking help,' and of 'close friendship [Friedkin, 1990].' Gould found solidarity within insurgent ranks to be positively impacted when pre-existing informal ties reinforced formal, organizational ties.

Even fewer attempt to mathematically model the implications of multiplex relations upon relationship strength. Renfro numerically estimates the influence potential of such bonds via a social closeness function—the stronger the bond, the greater the value of social closeness. Clark [2005] used a normalized version of the multidimensional centrality measures between relational graphs—originally described by Bonacich et al. [2004]—as a proxy for contextual weighting. Application of this measure, however, is dependent upon symmetric, unvalued relationship data that describes a connected graph. In general, this area poses several opportunities for investigating trade-offs between aggregate and independent analyses of contextual relationships, as well as weighting techniques to facilitate aggregation as needed.

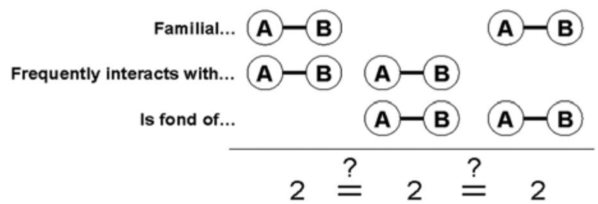

Figure 3: Improper Multiplex Aggregation

Another related advancement is detailed in Carley et al., which proposed an innovative concept describing a composite network that incorporates the multi-dimensionality of interpersonal relations via a meta-matrix. The meta-matrix concept is based upon the premise that network dynamics are functions of (1) the social structure, (2) the distribution of knowledge and information, (3) the interrelations between domains of knowledge, and (4) the distribution of work and requirements [Carley et al., 2002, pg. 83]. These network-related aspects of an organization within the meta-matrix construct serves as input into an agent-based network simulation, which evaluates the organization's ability to perform tasks, communicate effectively, and so forth [Carley et al., 2002].

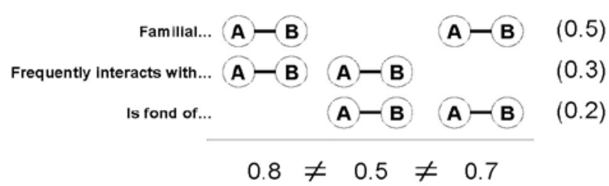

Figure 4: Potential Aggregation Scheme

The lack of previous studies may also be attributed to the complexity encountered when dealing with multiplex networks. Interestingly, Wasserman and Faust recommend that commonly used centrality and prestige measures be calculated for each relation separately and recommend against aggregating the relations into one sociomatrix [Wasserman and Faust, 1994, pg. 219]. Although rationale for this is not provided, the answer is likely the loss of information incurred when merely combining occurrences of links among relations.

For example, consider the three possible instances within which two individuals can share relations in two out of three contexts (Figure 3). Assuming that the contexts were 'familial,' 'frequently interacts with,' and 'is fond of,' one could posit examples where all of these possibilities would yield different strengths of relationships. However, a simple summation of dichotomous occurrences results in identical 'strengths' and is therefore likely insufficient to capture or infer the strength of a relationship based upon multiplex data.

Nonetheless, when two people interact, regardless of the value of the relationship's strength or a means to quantify it, we assume that both actors are cognizant of the underlying contexts that prevail and make their relationship either strong or tenuous. Consequently, social network measures applied to each of the networks or contexts independently will also fail to capture the combined effect due to the multiplexity inherent within the relationships. This suggests that, prior to determination of centrality, prestige, and so forth, an appropriate aggregation of contexts would be analytically prudent.

One potential means could comprise a weighted function, based upon how the actors internal to the network of interest place importance upon each context. A notional example of this is illustrated in Figure 4. Of course, one could ask the question "Is a familial link equivalent in strength to a tie that shares both the bonds of fondness and frequent interaction, since the weighted sum in either case equals 0.5?" The most likely answer is 'it depends,' and therefore remains an area ripe for further research.

Seeking to improve upon the available theoretical and methodological approaches used to analyze layered social networks, a series of approaches were developed to investigate various aspects of social networks where multiplex information is available [cf. Hamill, 2006]. The layered concept primarily provides a means to (1) derive a measure of relationship strength and (2) offer insight into potential courses of action that may increase the fragility of the target network or disrupt it entirely through the use of information operations.

It should also be noted, however, that methods attempting to measure strength of ties are generally criticized when applied to non-cooperative networks, such as terrorist organizations, in that increasing sophistication of analysis methodologies "may still not yield a more useful map" towards understanding the underlying network behavior [Fellman and Wright, 2003, pg. 5]. Nonetheless, ignoring this type of information automatically presumes all interpersonal ties are identical. When it is important to know not only who may successfully be exploited, but which interpersonal relationships play predominant roles within the efficacy of IO courses of action, assuming identical ties is ineffective.

Several methods (some of which apply contextual weights) to combining such information and their analytical implications are proposed in [Hamill, 2006]. When weights are applied, these values correspond to the relative importance the target network in toto places upon a particular relationship context. As the majority of this data is unlikely to be directly measurable, expert opinion familiar with the culture, indoctrination procedures, and institutional foundations will always play a vital role in providing guidance regarding the weights.

Approaches involving contextual weights lend themselves to additional analyses. For example, if PSYOP is applied to one or more layers, but not necessarily all of them, investigation of how these weights may change over time and the affect upon the network performance and exchange of influence (or power, or status, etc.) in response to these external forces—courses of action—are performed.

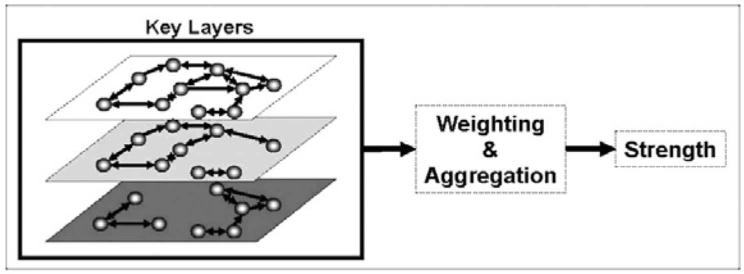

Figure 5: Layer Aggregation and Strength

The incorporation of a dynamic weighting scheme may also help ascertain the impact of information operations upon the network relationships. For example, compare the bottom layer to the top two in Figure 5. If information operations marginalized the weight of the top two layers from the individuals' point of view, a fissure between the network members may be observed. Therefore, despite the ability to measure exactly how much each context contributes to the strength of interpersonal relationships, the sensitivity of a given network to perturbations of the weight set, and the subsequent impact upon associated measures can be explored. As today's terrorist organizations are increasingly multi-cultural, extensions allowing for individual-specific weight sets should also be examined. This quest to quantify the nature of a network's links will (hopefully) lead us to a better understand of the resultant flow of information and influence among its members.

The Ebb and Flow of Influence

The seminal work of French described the then–current theory of social power and analyzed and addressed some of its limitations. In the course of his work, French defined "the basis of interpersonal power… as the more or less enduring relationship between (two individuals) A and B which gives rise to power" [French, 1956, pg. 183]. He then described five bases for power: attraction, expert, reward, coercive, and legitimate [French, 1956, pg. 183-4]. In examining the impact of peer group influence upon opinion formation, Friedkin and Cook's interpretation of French's work was that "[French] first proposed that social influence was a finite distributed resource" [Friedkin and Cook, 1990, pg. 130]. Within the context of OR methodologies, Renfro postulated that influence was analogous to a commodity flowing through a (social) network [Renfro, 2001, pg. 80-1]. These and other works such as the network flow centrality measure developed by Freeman et al., substantiate the modeling the flow of influence as a commodity within a network model.

Measurement of influence in the context of social network analysis (SNA) is "based upon the importance of relationships among interacting (individuals)" [Wasserman and Faust, 1994, pg. 4]. Additionally, one of the underlying principles of SNA is that "… individuals view the network structural environment as providing opportunities for or constraints on individual action" [Wasserman and Faust, 1994, pg. 4]. This implies individuals take into account opinions of those socially close, or in positions of authority, for example, when faced with a decision point.

There are a variety of examples in SNA literature that investigate and attempt to measure this influence [cf. Frank and Yasumoto, 1988, Friedkin and Cook, 1990, among others]. A predominant concentration of research in this area deals with determining what conditions, both internal (via the network structure and connectedness of individuals) and external (via the outside influences or requirements for a group-supported decision), are required to bring a group of individuals to agreement upon a group decision.

Friedkin and Cook discuss social influence in the context of interpersonal relations within a network, and their subsequent role regarding the interpersonal influence required to enable ". . . the process of (group) opinion formation" [1990, pg. 122]. This process utilizes network models to "… deal with the attainment of collective agreements…, usually beginning with a network of fixed and discrepant opinions" [Friedkin and Cook, 1990, pg. 122]. Modeling the processes of "interpersonal negotiation" and the subsequent change in individual opinions form the "unique theoretical thrust of network models of social influence …" [Friedkin and Cook, 1990, pg. 122-3]. The resulting models essentially attempt to describe the dyadic interaction required to transform a network of individuals with discrepant opinions into a network where the individuals' opinions have coalesced, at least to some degree. Similar concepts in social network literature based upon an exchange of influence between individuals include contagion (of behavior) (Leenders [2002]) and diffusion (the rate of acceptance of innovative and possibly risky ideas or behavior) [Valente, 1996].

Just as there are many network model formulations within the OR domain, numerous formulations and approaches exist within the study of social network modeling. Amblard and Deuant [2004] studied the propagation of extremist opinions throughout a variety of small-world networks. Their results suggest "… a critical level of connectivity and some disorder in the network (is necessary) in order for extreme opinions to invade a population …" [Amblard and Deuant, 2004, pg. 738].

However, this phenomenon is not necessarily confined to small-world networks. As Buchanan states the "infectious movement of desires and ideas from mind to mind is even the basis of a new theory of advertising known as permission marketing" [Buchanan, 2002, pg. 160-1]. Essentially, this connotes the flow of influence propagating through a general populous, which may not necessarily be a small-world network in the classical sense. This is an important point because not all organizations may naturally evolve as small-world networks; ultimately, influence will inevitably flow regardless of the underlying network structure [Renfro, 2001].

Beginning with French's influential work, it is clear that the social science research and theory liken the interaction between two individuals or groups to that of a commodity that flows between them. Operations Researchers and Social Scientists generally apply network models differently, a key difference being that social science models tend to be descriptive, while OR models tend to be both descriptive and prescriptive, where appropriate. Descriptive models, in general, attempt to describe how a process or system works via underlying relationships and behaviors. The focus of prescriptive models is improved decision making by attempting to describe the best or optimal solution of a given system [Clemen, 1996, pg. 14]. Oftentimes, the process of obtaining a prescriptive model requires an understanding of the underlying processes or systems inherent to the decision problem and therefore results in a descriptive model as a by-product.

Numerous modeling techniques within the SNA literature rely solely upon a single, presumably dominant, underlying context to study influence within a social network. If the simultaneous accommodation of multiple contexts is desired, the classic SNA techniques will almost certainly misidentify the key relationships of interest within a target network. Consequently, taking advantage of the strengths of both the SNA and OR modeling techniques offers new opportunities to improve our understanding of adversarial organizations. Unfortunately, simply obtaining the information necessary to characterizing adversarial networks presents a number of challenges.

Challenges of Network Data and SNA

Methods traditionally used to collect sociometric data include questionnaires, interviews, observations, archival records, experiments, and others; this implies that data sets comprise populations rather than subsets of them [Wasserman and Faust, 1994, pg. 45]. Granovetter notes "It is clear why network methods have been confined to small groups: existing methods are extremely sensitive, in their practicality, to group size because they are population rather than sampling methods" [Granovetter, 1976, pg. 1287-8].

Due to the potential N (N-1) number of directed ties between N individuals, collecting complete and accurate data on large populations is costly and problematic. Further, Granovetter argues that such studies can only make implicit connections between the nature of the data collected and the nature of the true population from which the data came. Ultimately, "… we are left guessing about the representativeness of the patterns of social relations found" [Granovetter, 1976, pg. 1288]. Viewing interpersonal relationships as multidimensional, as opposed to the generally single dimensional assessment, offers a means to improve upon existing models of social networks.

Although this, and other, research efforts assume that the data required for the methodologies presented is available and known with certainty, the mathematical nature of OR techniques permit the investigation of relaxations to this assumption via an array of sensitivity analyses.

One other potential limitation underlying social network research in general is the inherently static analysis of inevitably dynamic networks. It is certain that over time, some individuals may change their opinions or strategies, relationships evolve and devolve, and the overall social network structure changes due to recruitment of new individuals, new opportunities for interaction, and departures from the network. However, given the nature of available intelligence information and the near-term focus to which these techniques are amenable, it could be assumed that key changes in the network are primarily due to the actions or influence imposed upon it. Other efforts are pursuing the capability to simulate dynamic network behaviors, Carley [2003], for example; however, this can also require collecting a great deal more information that may or may not be available.

Unless the individuals that comprise the population are known with certainty, how representative the sample will be of the true population will always remain in question. For example, Tsvetovat and Carley [2005] have estimated Al Qaeda membership to be as high as 120,000. Even if such an extensive network could be mapped, it is likely that the magnitude would leave many current analysis capabilities computationally intractable. Hence, samples or subsets of the true networks comprise currently available data sets.

Other issues pervading network data that describes non-cooperative networks includes missing data and potential structural bias, as a result of the data gathering processes available. In a sociological or anthropological context, truly capturing information regarding a relationship between two individuals requires interviewing both [Stork and Richards, 1992, pg. 194]. When dealing with extremists, unless both individuals are in custody and amenable to truthful interviews, this is a difficult process dependent upon the skills of the interviewer and the interviewee, as well as some degree of luck. As a result, analysis must be performed on incomplete data.

Summary

When it comes to affecting networks of people in some way, in our context via information operations that may involve kinetic and non-kinetic means, the evaluation of potential target sets and courses of action must be accomplished. The evaluation of multiplex relations, in conjunction with other mathematical techniques, may be used to develop or evaluate the efficacy of proposed influence courses of action.

The overarching goal of this research area is to use new combinations of modeling techniques to improve our understanding of potential behavioral patterns that belie the target network, and their reactions to information operations imposed upon them. Subsequently, such understanding may help develop improved courses of action to effectively achieve a specified change in behavior in one or more actors within the target network.

We've briefly presented a number of methodologies and theories dealing with primarily open and cooperative social networks. The variety of Operations Research, Sociological, and Behavioral Theory efforts, all provide the bases for understanding relationship contexts and their affect upon the potential for interpersonal action (i.e. strength). The simple act of constructing these methodologies has brought economic, psychological, and other genres of study together with focused background investigation of various terrorist organizations.

The overall goal is realization of new and useful theory, and concomitant methodologies, describing and analyzing social networks of non-cooperative organizations. Given the improved understanding and insights provided by this developing research area, decision makers can be offered better courses of action seeking to achieve a target influence, perception, or outcome to one or more actors within the network through either direct or indirect means. These activities will then inevitably act as a forcing function to better understand and 'know' the enemy, providing new means to stifle their evolution, or eliminate them entirely.

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